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TIME VARYING FAILURE RATES IN RELIABILITY AND MEAN TIME TO SYSTEM FAILURE (MTSF) OF A SUMMER AIR CONDITIONING SYSTEM BY USING ALGEBRAIC METHOD

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ABSTRACT

In this paper, theauthors deal with the reliability and MTSF evaluation of a Complex continuous system in a summer air conditioning system for a place in hot and dry weather. In the considered system, two air dampers AD¬1, AD2 ; two airs filters AF1 , AF2 ; two cooling coils CC1,CC2 ; and two adiabatic humidifies AH1, AH2 have been taken in parallel redundancy to improve the reliability.A water eliminator weis attached with these components by some pipes Pi's. The object of the system is to supply cooled air for selected place. It has been assumed that the failure rates for various components of the complex system follow arbitrary distribution and there is no service facility to repair the considered system. By using Algebraic Method solve the mathematical model of this problem.

KEYWORDS: Algebraic Method, Markovian Process, Weibull or Exponential Distribution, Steady State Behavior.

INTRODUCTION

In the recent past, quite a good number of studies have been carried out by earlier researchers [1,3,5,6,8] to compute reliability expression for different complex systems by Boolean function Technique but a very little work has been done in evaluating the reliability of a complex system by using Algebraic Method. Keeping the above facts in view, in this paper, we have computed the reliability and MTSF of a complex system by using Algebraic Method. This method is based on the well-known theorem of summation of probabilities of compatible events. In this paper, "a summer air conditioning system for a place in hot and dry weather" is considered to evaluate the reliability and mean time to failure. Such systems are used for hot and dry outdoor conditions like Nagpur, Delhi, Bhopal and other places. The comfort conditions required in an air conditioned space are 24^o C DBT (Dry Bulb Temperature) and 60% RH (Relative Humidity). The atmospheric conditions at Nagpur in summer are 40.5^o CDBT and 20% R.H. The arrangement of equipment's required for an ordinary system is represented in Fig-1 and adiabatic saturation arrangement is represented in Fig-2. We have considered a particular system to improve the reliability. The block diagram of considered complex system has been shown in Fig.-3. In the considered system, two air dampers AD_1, AD_2 ; two airs filters AF_1 , AF_2 ; two cooling coils CC_1,CC_2 ; and two adiabatic humidifies AH_1, AH_2 have been taken in parallel redundancy to improve the reliability. A water eliminator WE is attached with these components by some pipes P_i's. The object of the system is to supply cooled air for selected place. It has been assumed that the failure rates for various components of the complex system follow arbitrary distribution and there is no service facility to repair the considered system. Reliability of the complex system has been computed when the failure rates of various components follow either weibull or exponential distribution. MTSF, an important parameter of reliability has also been evaluated for exponential distribution. A numerical example along with tables and graphs has been appended at the end to highlight the important results.

ASSUMPTIONS

1. Initially, all components of the system are good.
2 The states of all components are statistically independent.
3. Failure rates of all the components are arbitrary. There is no repair facility.

NOTATIONS

$\wedge$ / $\vee$: Conjunction / Disjunction.

x_i' : Negation of x_i, for i=1 through 18.

x_i : $\begin{cases} 0, \text{ in bad states.} \\ 1, \text{ in good states (i=1 to 18).} \end{cases}$

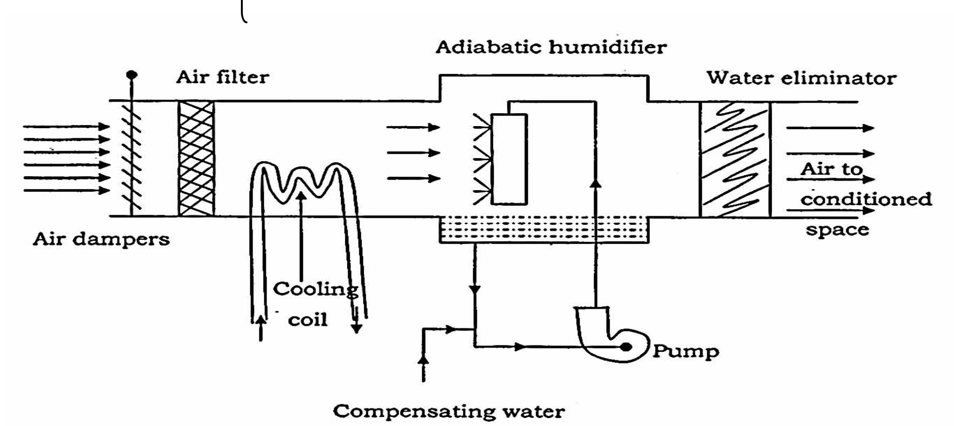

Figure-1(Summer air Conditioning System for Hot and Dry Weather)

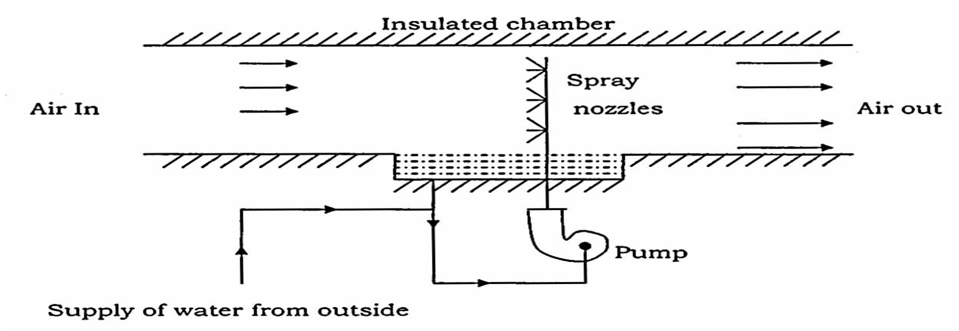

Figure-2 (Adiabatic saturation arrangement)

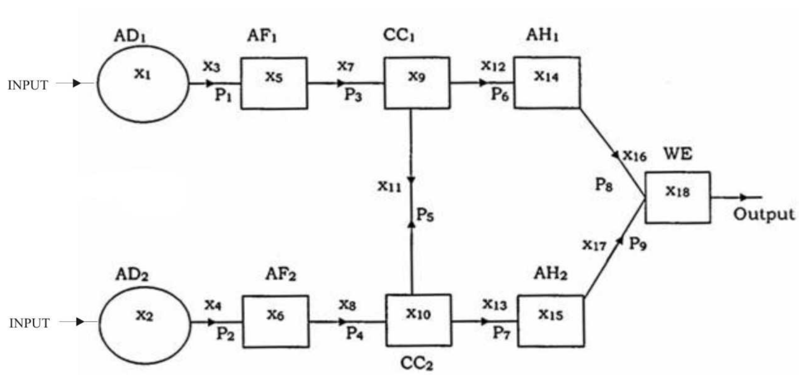

Figure-3(Block Diagram of Considered System)

FORMULATION OF MATHEMATICAL MODEL

By using Boolean function technique, the conditions of capability for the successful operation of the considered system in terms of logical matrix are expressed as :

$$f(x_1, x_2, x_3, \ldots\ldots\ldots, x_{18}) = \begin{vmatrix} x_1 & x_3 & x_5 & x_7 & x_9 & x_{12} & x_{14} & x_{16} & x_{18} & & \\ x_1 & x_3 & x_5 & x_7 & x_9 & x_{11} & x_{10} & x_{13} & x_{15} & x_{17} & x_{18} \\ x_2 & x_4 & x_6 & x_8 & x_{10} & x_{13} & x_{15} & x_{17} & x_{18} & & \\ x_2 & x_4 & x_6 & x_8 & x_{10} & x_{11} & x_9 & x_{12} & x_{14} & x_{16} & x_{18} \end{vmatrix}$$

$$= \begin{vmatrix} H_1 \\ H_2 \\ H_3 \\ H_4 \end{vmatrix} \quad \text{(say)}$$

Now using the theorem of summation of probabilities of compatible events, viz.

$$P_r\left\{\bigcup_{i=1}^{n} H_i\right\} = \sum_{i=1}^{n} P_r(H_i) - \sum_i \sum_j P_r(H_i \cap H_j) + \sum_i \sum_j \sum_k P_r(H_i \cap H_j \cap H_k) +$$
$$\ldots\ldots\ldots + (-1)^{n-1} P_r(H_1 \cap H_2 \cap H_3 \cap \ldots\ldots\ldots\ldots \cap H_n),$$

The probability of the successful operation of the function f, i.e. reliability of the power supply, is given by

$$R_s = P_r(f = 1) = P_r\left[\bigcup_{i=1}^{n} H_i\right]$$

where$R_i^{'}$s are the reliabilities of the components corresponding to states $X_i^{'}$s respectively.

PARTICULAR CASES

Case I: If the reliability of each component of the complex system is of the same numerical measure i.e. of R. In this case, we get

$$R_s = 2R^9 + 2R^{11} + 2R^{18} - 2R^{14} - 2R^{15} - R^{17} \quad (3)$$

Case II: if the Failure Rate Follows a Weibull Distribution

In this case, let λ_i be the failure rate of the component at the stage X_i, respectively. Then from equation (2), reliability of the complex system at an instant t, is given by:

$$\begin{aligned} R_{SW}(t) = {} & \exp(-\alpha_1 t^n) + \exp(-\alpha_2 t^n) + \exp(-\alpha_3 t^n) + \exp(-\alpha_4 t^n) \\ & +2\exp(-\alpha_5 t^n) - \exp(-\alpha_6 t^n) - \exp(-\alpha_7 t^n) - \exp(-\alpha_8 t^n) \\ & - \exp(-\alpha_9 t^n) - \exp(-\alpha_{10} t^n) \end{aligned} \quad (4)$$

where n is a positive parameter and α_i's are given by:

$\alpha_1 = \lambda_1 + \lambda_3 + \lambda_5 + \lambda_7 + \lambda_9 + \lambda_{12} + \lambda_{14} + \lambda_{16} + \lambda_{18}, \alpha_2 = \alpha_5 - \lambda_2 - \lambda_4 - \lambda_6 - \lambda_8 - \lambda_{12} - \lambda_{14} - \lambda_{16}$

$\alpha_3 = \lambda_2 + \lambda_4 + \lambda_6 + \lambda_8 + \lambda_{10} + \lambda_{13} + \lambda_{15} + \lambda_{17} + \lambda_{18}, \alpha_4 = \alpha_5 - \lambda_1 - \lambda_3 - \lambda_5 - \lambda_7 - \lambda_{13} - \lambda_{15} - \lambda_{17}$

$$\alpha_5 = \sum_{i=1}^{18} \lambda_i \ , \ \alpha_6 = \alpha_5 - \lambda_2 - \lambda_4 - \lambda_6 - \lambda_8 \ , \ \alpha_7 = \alpha_5 - \lambda_{11}$$

$$\alpha_8 = \alpha_5 - \lambda_{13} - \lambda_{15} - \lambda_{17} \ , \ \alpha_9 = \alpha_5 - \lambda_{12} - \lambda_{14} - \lambda_{16} \ , \ \alpha_{10} = \alpha_5 - \lambda_1 - \lambda_3 - \lambda_5 - \lambda_7$$

Case III: if the Failure Rates Follow on Exponential Distribution:

In this case, reliability of the complex system at an instant t can be find out by putting n = 1 in equation (4), as given below

$$\begin{aligned} R_{SE}(t) & = [R_{SW}(t)]_{n=1} \\ & = \exp(-\alpha_1 t) + \exp(-\alpha_2 t) + \exp(-\alpha_3 t) + \exp(-\alpha_4 t) + 2\exp(-\alpha_5 t) \\ & \quad - \exp(-\alpha_6 t) - \exp(-\alpha_7 t) - \exp(-\alpha_8 t) - \exp(-\alpha_9 t) - \exp(-\alpha_{10} t) \end{aligned} \quad (5)$$

and the expression for MTSF, in this case is given by

$$\begin{aligned} MTSF & = \int_0^\infty [R_{SE}(t)]dt \\ & = \frac{1}{\alpha_1} + \frac{1}{\alpha_2} + \frac{1}{\alpha_3} + \frac{1}{\alpha_4} + \frac{2}{\alpha_5} - \frac{1}{\alpha_6} - \frac{1}{\alpha_7} - \frac{1}{\alpha_8} - \frac{1}{\alpha_9} - \frac{1}{\alpha_{10}} \end{aligned} \quad (6)$$

NUMERICAL COMPUTATIONS

For Reliability

Setting $\lambda_i = 0.01$ (for i = 1,2, ..., 18) and n = 2 in equation (4) and (5), we get the equations for $R_{SW}(t)$ and $R_{SE}(t)$ as follows:

$$R_{SW}(t) = 2\exp(-0.09t^2) + 2\exp(-0.11t^2) + 2\exp(-0.18t^2) - 2\exp(-0.14t^2)$$
$$-2\exp(-0.05t^2) - \exp(-0.17t^2)$$

$$R_{SE}(t) = 2\exp(-0.09t) + 2\exp(-0.11t) + 2\exp(-0.18t) - 2\exp(-0.14t)$$
$$-2\exp(-0.05t) - \exp(-0.17t)$$

Now with the help of above two equations we get the table-1, which shows the changes of $R_{SW}(t)$ and $R_{SE}(t)$ with respect to time t.

For MTSF

Setting $\lambda_i = \lambda$ (for i = 1,2, 3, ..., 18) in equation (6), we get the equation

$$MTSF = \frac{2}{9\lambda} + \frac{2}{11\lambda} + \frac{2}{18\lambda} - \frac{2}{14\lambda} - \frac{2}{15\lambda} - \frac{1}{17\lambda} = \frac{1}{\lambda}$$

and with the help of this equation we get table-2, which shows the changes in MTSF with respect to failure rate λ.

CONCLUSIONS

It can be written after seen the graphs both reliabilities should decreases with respect to time t and initially both are equal to 1. MTSF also decreases as λ increases.

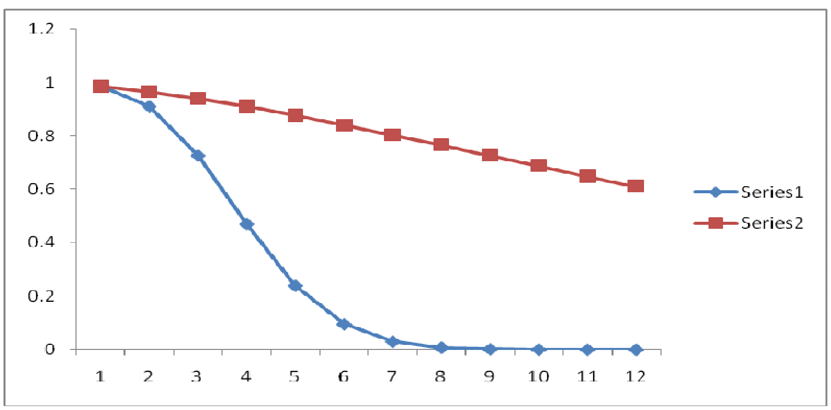

Fig.-1 (Reliability with Respect to time)

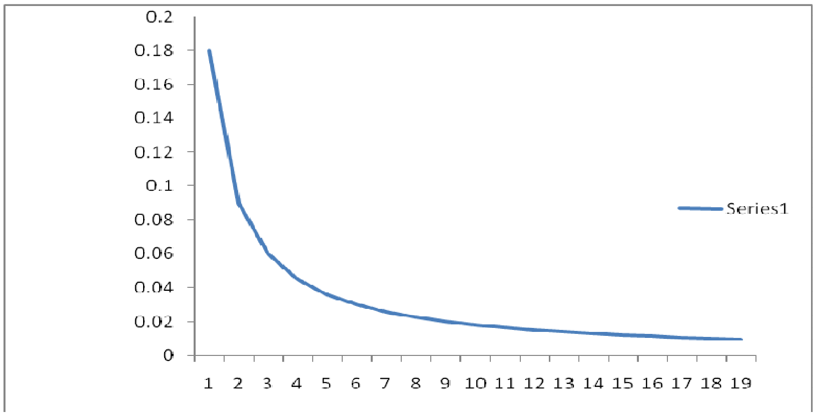

Fig.-2 (Mean time to System Failure with Respect to time)

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